The long-time tails of granular fluids

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自由冷却状態における粉体ガスでの粒子自己速度相関関数及びストレスの相関相関は衝突回数でスケールされた時間 τ 及び空間次元 d に対して $\tau^{-d/2}$ というロングタイムテールがあることを明らかにし、一方熱流の自己相関関数は指数関数的に減衰することを発見した。

It is crucially important to clarify whether or not there are the long-time tails in the auto-correlation functions to know the rhelogical properties of fluids. In fact, it is known that the transport coefficients in two-dimensional systems diverge in the thermodynamic limit[1], because of the long-time tail decaying as $t^{-d/2}$ with the time t and the spatial dimension d in elastic gases.[2] Thus, if there are the long-time tails for granular fluids, we may need to change the transport coefficients determined by inelastic Boltzmann equation[3] and Enskog equations.[4, 5] Recently, Kumaran[6] indicates the suppresses of the long-time tail in sheared granular flows as $t^{-3d/2}$, while Ahmad and Puri[7] suggest the existence of the long-time tail as τ^{-1} with the scaled time τ by the collision frequency in freely cooling granular gases from their two-dimensional simulation. Therefore, we need to clarify what is the true story of the long-time tail and whether the transport coefficients exist in two-dimensional granular fluids.

Let us consider the granular gas characterized by a constant restiution coefficient e which is less than 1. The current auto-correlation functions are introduced as

$$
\begin{aligned}
C_D(\tau) &= \lim_{V\to\infty}\frac{1}{V}\sum_{i=1}^{N}\langle v_{ix}(0)v_{ix}(t)\rangle, &(1)\\
C_\eta(\tau) &= \lim_{V\to\infty}\frac{1}{V}\langle J_\eta(0)J_\eta(t)\rangle, &(2)\\
C_\lambda(\tau) &= \lim_{V\to\infty}\frac{1}{V}\langle J_\lambda(0)J_\lambda(\tau)\rangle. &(3)
\end{aligned}
$$

where $J_\eta(t)$ and $J_\lambda(t)$ are the kinetic part of the stress and the heat flux defined as $J_\eta(t) = \sum_i mv_{ix}v_{iy}$ and $J_\lambda(t) = \sum_i \frac{1}{2}\left(mv_i^2 - (d+2)T\right)v_{ix}$.

Our approach is parallel to that for elastic gases developed by Ernst et al.[2] in which the fluctuations are characterized by linearized hydrodynamics. The correlation functions, thus, are

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functions of the viscosity η^*, the diffusion constant D^*, the heat condutivity λ^*, the cooling rate due to inelastic collisions ζ^* all of which are nondimensionalized quantities associated with the intial temperature T_0, the first Sonine coefficient a_2 and the characteristic length l_H estimaed by the ratio of the thermal velocity to the collision frequency.

The result of $C_D(\tau)$ is given by

$$C_D(\tau) \simeq \frac{T_0}{dml_H^d}\left\{(d-1)\left(\frac{1}{2\pi(\eta^*+2D^*)\tau}\right)^{d/2}+\left(\frac{1}{4\pi(a+D^*)\tau}\right)^{d/2}\right\}, \tag{4}$$

where $a=(d-1)\eta^*/d+1/(d\zeta^*)$. Thus, we confirm that the asymptotic behavior $C_D(\tau)$ decays as $\tau^{-d/2}$ which is consistent with the simulation by Ahmad and Puri.[7] Similarly, the result of $C_\eta(\tau)$ becomes

$$C_\eta(\tau) \simeq \frac{T_0^2(1+a_2)}{d(d+2)l_H^d}\left((d^2-2)\left(\frac{1}{4\pi\eta^*\tau}\right)^{d/2}+2d\left(\frac{1}{2\pi(\eta^*+2a)\tau}\right)^{d/2}+2\left(\frac{1}{8\pi a\tau}\right)^{d/2}\right), \tag{5}$$

which also has the long-time tail obeying $\tau^{-d/2}$.

On the other hand, $C_\lambda(\tau)$ does not have any long-time tail. The explicit form is given by

$$C_\lambda(\tau) \simeq -\frac{\pi(d+2)^2(d-1)T_0^3A}{2d^3m\zeta^{*2}l_H^d}\left(\frac{(d-1)e^{-\zeta^*\tau}}{(2\pi(\eta^*+2a)\tau)^{\frac{d+2}{2}}}+\frac{2de^{-\zeta^*\tau}}{(8\pi a\tau)^{\frac{d+2}{2}}}\right), \tag{6}$$

where $A=2(d-1)+a_2(9d-10)$. The absence of the long-time tail in $C_\lambda(\tau)$ is the result of the lack of energy conservation law.

The validity of the theoretical prediction has been confirmed by the direct simulation of the two-dimensional cooling process of granular gases. It is interesting that $C_D(\tau)$ obeys τ^{-1} in both the early stage and the late stage, but the deviation is large in the middle of the cooling process. The details will be published in elsewhere.

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